Homework 8  Matrices

1. Given the linear system below :
   \[ \begin{align*}
   v + 2w - y + z &= 1 \\
   3y + x - z &= 2 \\
   x + 7y &= 1
   \end{align*} \]
   
   a. Find the Coefficient Matrix \( A \), the State variable vector \( X \) and the input constant vector \( B \)
   
   b. What are the sizes of \( A \), \( X \) and \( B \)
   
   c. Write the system as \( A \times X = B \)

2. Prove:
   
   a. For each \([n \times n]\) matrix \( A \), \( A^T \times A \) is symmetric
   
   b. If \( A \) is a symmetric matrix, then so are \( A^2 \) and \( 2A^2 - 3A + I \)

3. A complex number \( z_1 = x_1 + jy_1 \) may be rotated by an angle \( \theta \) (measured counterclockwise) in the complex plane by forming the product \( z_2 = z_1 e^{j\theta} \)

   The real and imaginary parts of \( z_2 \) are
   
   \[ \begin{align*}
   \text{Re}(z_2) &= x_2 = \cos(\theta)x_1 - \sin(\theta)y_1 \\
   \text{Im}(z_2) &= y_2 = \sin(\theta)x_1 + \cos(\theta)y_1
   \end{align*} \]

   If the real and imaginary parts of \( z_1 \) and \( z_2 \) are organized into vectors

   \[ \begin{pmatrix}
   \text{Re}(z_1) \\
   \text{Im}(z_1)
   \end{pmatrix} \quad \text{and} \quad \begin{pmatrix}
   \text{Re}(z_2) \\
   \text{Im}(z_2)
   \end{pmatrix} \]

   Find a rotation matrix \( R(\theta) \) and Show that the rotation can be carried out with the matrix-vector multiplication

   \[ z_2 = R(\theta) \times z_1 \]

4. Solve circuit 2 and obtain all node voltages and currents in the circuit.
5. Suppose that in circuit 1, the battery was not 5 Volts but variable. By performing an appropriate looping function, plot lamp current as a function of battery voltage and comment on the result. Vary the battery voltage between 0 and 10 volts.

Figure 1: Circuit 1

Figure 2: Circuit 2