HOMEWORK 6
SOLUTION

I. $\mathbf{y(n+2) - 2y(n+1) - 3y(n) = 0}$

Initial Conditions: $\mathbf{y(1) = 5}$
$\mathbf{y(2) = 19}$

1. Order 2.
2. Assume $y_H(n) = \lambda^n$

$$\lambda^{n+2} - 2\lambda^{n+1} - 3\lambda^n = 0$$
$$\lambda^2 - 2\lambda - 3 = 0 \qquad \text{(Characteristic Polynomial)}$$
$$(\lambda + 1)(\lambda - 3) = 0$$

$$\left.\begin{array}{l}\lambda_1 = -1\\ \lambda_2 = 3\end{array}\right\} \quad \text{2 distinct roots}$$

$$y_H(n) = C_1(\lambda_1)^n + C_2(\lambda_2)^n$$

$$\boxed{y_H(n) = C_1(-1)^n + C_2(3)^n}$$

$$y(1) = 5 = C_1(-1)^1 + C_2(3)^1 = -C_1 + 3C_2 \qquad (1)$$

$$y(2) = 19 = C_1(-1)^2 + C_2(3)^2 = C_1 + 9C_2 \qquad (2)$$

Adding 1 and 2:

$$12C_2 = 24$$

$$\boxed{C_2 = 2}$$

$$-C_1 + 3C_2 = 5$$
$$C_1 = 3C_2 - 5$$
$$C_1 = 6 - 5$$

$$\boxed{C_1 = 1}$$

$$y(n) = C_1(-1)^n + C_2(3)^n$$

$$\boxed{y(n) = (-1)^n + 2(3)^n}$$

II. $\mathbf{y(k) - 7y(k-2) + 6y(k-3) = 0}$

Initial Conditions:

$$\mathbf{y(0) = 2}$$
$$\mathbf{y(1) = 1}$$
$$\mathbf{y(2) = 3}$$

Assume: $y_H(k) = \lambda^k$

$$\lambda^k - 7\lambda^{k-2} + 6\lambda^{k-3}$$
$$\lambda^k (\lambda^3 - 7\lambda + 6) = 0$$

$$\boxed{\lambda^3 - 7\lambda + 6 = 0}$$ Characteristic Polynomial

Checking if $\lambda = 1$ is a root:

$$(1)^3 - 7(1) + 6 = 0$$

$$\Rightarrow \quad \boxed{\lambda_1 = 1 \text{ is a root}}$$

$$\lambda^3 - 7\lambda + 6 = (\lambda - 1)(\lambda^2 + \lambda - 6) = 0$$

$$\lambda^2 + \lambda - 6 = 0$$

$$\boxed{\lambda_2 = 2 \text{ and } \lambda_3 = -3 \text{ are roots of } \lambda^2 + \lambda - 6 = 0}$$

$$Y_H(k) = C_1(\lambda_1)^k + C_2(\lambda_2)^k + C_3(\lambda_3)^k$$

$$\Rightarrow \quad \boxed{y_H(k) = C_1(1)^k + C_2(2)^k + C_3(-3)^k}$$

Using initial conditions to find C_1, C_2 and C_3:

$$y(0) = C_1(1)^0 + C_2(2)^0 + C_3(-3)^0 = 2$$
$$y(1) = C_1(1)^1 + C_2(2)^1 + C_3(-3)^1 = 1$$
$$y(2) = C_1(1)^2 + C_2(2)^2 + C_3(-3)^2 = 3$$

$$\left.\begin{array}{c} C_1 + C_2 + C_3 = 2 \\ C_1 + 2C_2 - 3C_3 = 1 \\ C_1 + 4C_2 + 9C_3 = 3 \end{array}\right\} \Rightarrow \begin{array}{c} C_1 = 2 \\ C_2 = -0.2 \\ C_3 = 0.2 \end{array}$$

$$y_H(k) = 2(1)^k - 0.2(2)^k + 0.2(-3)^k$$

III. $\mathbf{y(k) - 3y(k-1) + 2y(k-2) = 4^k}$ **$k > 0$**

Initial conditions:

$$\mathbf{y(0) = 1}$$
$$\mathbf{y(1) = 2}$$

Finding homogeneous solution:
Homogeneous equation:

$$y(k) - 3y(k-1) + 2y(k-2) = 0$$

$$\lambda^k - 3\lambda^{k-1} + 2\lambda^{k-2} = 0$$
$$\lambda^{k-2}(\lambda^2 - 3\lambda + 2) = 0$$
$$(\lambda^2 - 3\lambda + 2) = 0 \qquad \text{Characteristic Polynomial}$$

Finding the roots:

$$(\lambda - 1)(\lambda - 2) = 0$$

$$\boxed{\begin{array}{c}\lambda_1 = 1 \\ \lambda_2 = 2\end{array}}$$

λ_1 and λ_2 are roots of $(\lambda^2 - 3\lambda + 2) = 0$

Homogeneous solution: $\boxed{y_H(k) = C_1(1)^k + C_2(2)^k}$

Finding particular solution:
Input signal = 4^k, It has the form of AM^k, then:

$$y_p(k) = BM^k \quad \text{find B}$$

$y_p(k)$ is a solution for the difference equation 1, then:

$$y_p(k) - 3y_p(k-1) + 2y_p(k-2) = 4^k$$

$$B\,4^k - 3B\,4^{k-1} + 2B\,4^{k-2} = 4^k$$
$$\not{4}^k(B - 3B\,4^{-1} + 2B\,4^{-2}) = \not{4}^k$$
$$B - \frac{3B}{4} + \frac{2B}{16} = 1$$
$$B = \frac{16}{8} = \frac{8}{3}$$
$$y_p(k) = \frac{8}{3}4^k$$
$$y_G(k) = y_p(k) + y_H(k)$$

$$y_G(k) = \frac{8}{3}4^k + C_1(1)^k + C_2(2)^k$$

Plugging in the general equation the initial conditions:

$$y(0) = 1 = \frac{8}{3}4^0 + C_1(1)^0 + C_2(2)^0$$

$$\boxed{\frac{8}{3} + C_1 + C_2 = 1}$$

$$y(1) = 2 = \frac{8}{3}4^1 + C_1(1)^1 + C_2(2)^1$$

$$\boxed{\frac{32}{3} + C_1 + 2C_2 = 2}$$

Solving the system of equations:

$$C_1 = \frac{16}{3}$$

$$C_2 = -7$$

$$y_G(k) = \frac{8}{3}4^k + \frac{16}{3}(1)^k - 7(2)^k$$