Atmospheric Temperature Retrieval

From the differential equation

\[ \frac{dI}{d\tau} = -(I - B(\tau)) \quad \text{with B.C: } I(0) = B(T_s) \]

we have \( I_{\text{TOA}}^k = B(T_s, \lambda_k) \exp(-\tau_k) + \int_0^{\tau_k} B(T'_{\tau_k}, \lambda_k) \exp(-(\tau_k - \tau')) d\tau' \) where \( k \) is the wavelength channel index. Here, the first term represents the surface contribution absorbed through the atmosphere and the second integral defines the emission of all atmosphere levels of optical thickness \( d\tau' \) at height \( \tau' \) which is then absorbed by the gas column above the layer.

It is convenient to recast in altitude coordinates so we need a relationship \( \tau_k' = \tau(z', \lambda_k) \).

Clearly, \( \alpha(z', \lambda_k) = N(z') \sigma(\lambda_k) = N_0 \sigma(\lambda_k) \exp\left(-\frac{z'}{H}\right) \) so

\[ \tau(z') = \int_0^{z'} \alpha dz' = N_0 \sigma H \left(1 - \exp\left(-\frac{z'}{H}\right)\right) = \tau_0 \left(1 - \exp\left(-\frac{z'}{H}\right)\right) \]

where \( \tau_0 = \int_0^{\infty} \alpha dz' = N_0 \sigma H \) is the full column optical depth of the full atmosphere.

The above Integral can then be recast as \( \int_0^{\infty} B(T(z'), \lambda_k) W(z', \lambda_k) dz' \) where the weight function is

\[ W(z', \lambda_k) = \frac{\tau(\lambda_k)}{H} \exp\left(-\frac{z'}{H}\right) \exp\left(-\tau(\lambda_k) \exp\left(-\frac{z'}{H}\right)\right) \].

Clearly, the spectral dependence enters through the spectral dependence of the column optical depth of the gas.

As discussed in class, we need a gas which mixes well with the atmosphere so that the above analysis will work. Luckily CO2 is well mixed with a nearly constant number mixing ratio of 370 ppm throughout the entire atmosphere. Therefore, it is important to calculate the column optical depth of CO2 to see which channels we will use.

The full plot is given below
The high resolution image is given below

The resolution of most High Spectral resolution sensors in the IR such as AIRS with 0.5 cm\(^{-1}\) resolution is capable of isolating each of these lines.
If we use the AIRS channels with 1.5 cm\(^{-1}\) resolution, (see below), the column AOD will provide a smooth transition from low AOD channels which see to lower altitudes with high AOD channels which only see the high altitude emission.
Using the AOD spectral dependance, we can plot the different weighting functions for each spectral channel. The results are plotted below and show the significant problems inherent in temperature retrieval since the weight functions have such dramatic overlap.

Clearly, we can not resolve temperature with high resolution. However, if we consider simply trying to retrieve temperatures over large vertical areas (assuming the temperature is constant), it is somewhat easier.

Imagine we partition the atmosphere into N layers \( L_n = [z_{n-1}, z_n] \) \( n = 1 : N \) where the temperature is constant for each layer and \( n=0 \) corresponds to the surface.

Then \( I_{\text{TOA}}^k = B_{0,k} W_{o,k} + \sum_{n=1}^{N} B_{n,k} W_{n,k} = \sum_{n=0}^{N} B_{n,k} W_{n,k} \) where \( B_{n,k} = B(T_n, \lambda_k) \) and the weight functions are \( W_{o,k} = \exp(-\tau_k) \) for the surface and \( W_{n,k} = \int_{z_{n-1}}^{z_n} W(z', \lambda_k) \, dz' \) for the different atmosphere levels.
The weights for the 20 wavelength bands are calculated for the surface level and 10 atmosphere levels.

Numerical Inversion

Step 1. Assume Temperature distribution $T_n \quad n = 0 : 10$

Step 2. Calculate $I_{TOA}^k = \sum_{n=0}^{N} B_n W_n, k$ where the coefficients are defined above.

Step #3 Convert to Brightness Temperature by Inverting Planck’s formula $I = \frac{c_1 \nu^3}{\exp\left(\frac{c_2 \nu}{T}\right) - 1}$

obtain $T_B = c_2 \nu \left[ \log(1 + \frac{c_1 \nu^3}{I}) \right]$

Step 4. Add Sensor Noise level as given in equivalent temperature noise. For example, I use $\Delta T_B = 0.1K$
Step 5. Convert back to TOA Irradiance $I_k^{\text{meas}}$ through Planck’s Function.

Step 6. Perform numerical Inversion to minimize $\sum_{l=1}^{n} \ln \left( I_l^{\text{meas}} - I_l^{\text{fit}} \left(T_{n}^{\text{fit}}\right) \right)^2$ using.

Results